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Frequency domain design and implementation of separable two dimensional Farrow structure

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***Abstract*— This paper gives an overview of the separable two dimensional transposed Farrow structure. We analyze a separable transposed implementation structure in terms of complexity. We also present a straightforward procedure to design the separable 2-D Farrow structure. The procedure is to apply the best known one dimensional frequency domain design method for polynomial-based filters in each dimension. In this way, all frequency domain requirements, such as passband ripple, stopband attenuation, passband and stopband edges are defined independently for each dimension.**

***Keywords*— capacitive sensor, 2-D digital filters, Farrow structure, Filter design**

I. Introduction

The Farrow structure has been used in many cases to build variable digital filters, with adjustable frequency characteristics [1]-[5]. In many signal processing applications it is required to determine signal samples at arbitrary positions between existing samples of a discrete-time signal. In these cases, it is beneficial to use polynomial-based interpolation filters. For these filters, an efficient overall implementation can be achieved by using a continuous-time impulse response $h_a(t)$ having the following properties **Error! Reference source not found.**], [2]; First, $h_a(t)$ is nonzero only in a finite interval $0 \le t \le NT$ with N being an integer. Second, in each subinterval $nT \le t < (n+1)T$, for $n = 0, 1, \ldots, N-1$, $h_a(t)$ is expressible as a polynomial of t of a given (low) order M. Third, $h_a(t)$ is symmetric with respect to $t = NT/2$ to guarantee phase linearity of the resulting overall system. The length of polynomial segments, T, can be selected to be equal to the input T_{in} or output T_{out} sampling interval, a fraction of the input or output sampling interval, or an integer multiple of the input or output sampling interval. The advantage of the above system lies in the fact that the actual implementation can be efficiently performed by using the Farrow structure [1] or its modifications [2], [3].

The original Farrow structure has been modified to 2-D case in order to build variable digital filters for applications mainly in digital image processing [4], [5]. For example, in image processing we are interested in two-dimensional interpolation in order to achieve better image resolution. In the literature, several two-dimensional interpolation methods and two-dimensional modifications of the Farrow structures with design methods have been proposed [4]-[6]. In [6] Shyu et al. presented an effective 2-D Farrow structure and the design of variable fractional delay filters suitable for digital image processing. The design method exploits the symmetric/antisymmetric relationship between filter coefficients. The structure of [6] is good option to implement nonseparable circularly symmetric low-pass variable fractional delay filters. In [4], Sankaran et al. presented 2-D Farrow structure suitable for separable case, which is used for nonuniform to uniform image resampling.

In this paper, we analyze separable 2-D Farrow filters in terms of implementation complexity. We also study frequency domain performance of the analyzed separable 2-D Farrow filter.

II. Separable 2-D Farrow Structure

As stated above, Sankaran et al. presented 2-D Farrow structure which is used for non-uniform to uniform image resampling [4]. The structure is separable, and allows one an independent design in each dimension. The structure of [4] is intended for efficient reduction of image resolution and each separable filter is given in the transposed form. In a sequel we analyze this separable transposed 2-D Farrow structure.

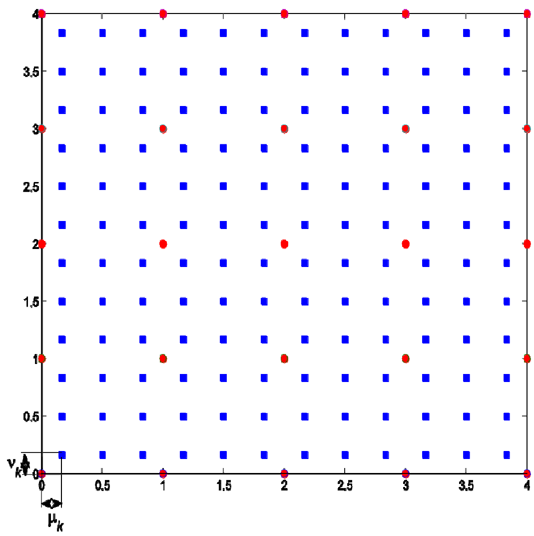

Fig. 1 Calculating fractional intervals.

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However, all conclusions can be drawn in a dual direct form which is more suitable for image interpolation.

As originally derived in [4], the separable transposed 2-D Farrow structure can be represented by following input/output relation:

$$y(s,t)=\sum_{m_1=0}^{M_1}\sum_{i_1=0}^{N_1} c_{m_1}(i_1)\sum_{m_1=0}^{M_1}\sum_{i_1=0}^{N_1} b_{m_1}(i_1)\cdot \\ \cdot\sum_k x(j_k,l_k)(\mu_k)^{m_1}(\vartheta_k)^{m_2}, \qquad (1)$$

where $x(j_k,l_k)$ is an input pixel, $y(s,t)$ is an output pixel, $c_{m_1}(i_1)$ are coefficients of the transposed Farrow structure in the first dimension, $b_{m_1}(i_1)$ are coefficients of the transposed structure in the second dimension, μ_k is the fractional interval in the first dimension, and ϑ_k is the fractional interval in the second dimension. Figure 1 illustrates how the input parameters μ_k and ϑ_k are calculated. The inner sum in (1) is represented by term *blockid* in Fig. 2 to refer to the square box formed between the integer intervals in the 2D grid. For each input sample, the integer block to which it belongs in the output grid is calculated from j_k, l_k. The sampling rate conversion is done in the accumulators similar to the 1D transposed Farrow structure. The accumulators are reset when $blockid_{k-1} \neq blockid_k$. All the accumulators are reset synchronously. Equation (1) can be represented by a two dimensional structure utilizing the transposed Farrow structure as shown in Fig. 3 where TF stands for the transposed Farrow structure [4], [15]. The transposed Farrow structure is shown in Fig 2. The structure shown in Fig. 3 is the separable transposed 2-D Farrow structure. The expanded form of the structure is given in Fig. 4.

The computational complexity of the separable 2-D transposed Farrow structure depends on polynomial order, and number of polynomial segments (filter length) in each direction. The computational load can be further decreased by exploiting symmetry/antisymmetry between coefficients.

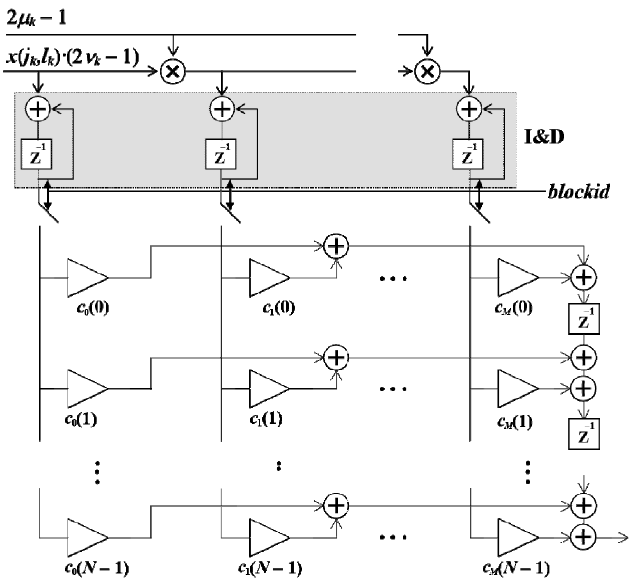

Fig. 2 The transposed Farrow structure (TF).

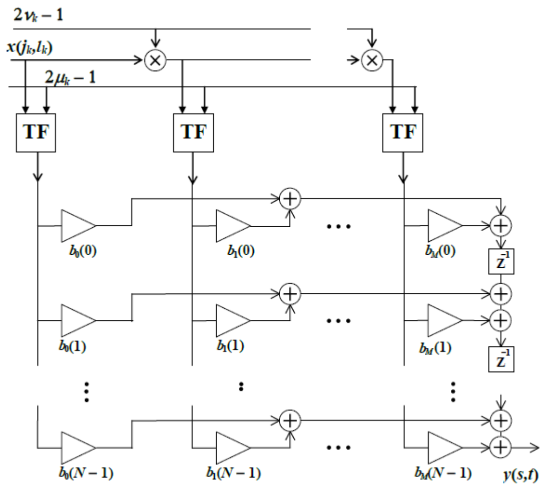

Fig. 3 Two-dimensional Farrow structure using TF blocks, where TF is the transposed Farrow structure shown in Fig 2.

The number of multipliers can be expressed as:

$$S=(M_1+1)\cdot N_1/2+M_1+(M_2+1)\cdot N_2/2+M_2 \qquad (2)$$

where M_1 and M_2 are filter orders, and N_1 and N_2 are filter lengths in each direction. We can see that the filter order M_i and filter length N_i can be selected independently, as it will be described in the next section.

III. Frequency Domain Analysis of Separable 2-DFarrow Structure

In this chapter we present a straightforward procedure to design the separable 2-D Farrow structure. The procedure is to apply the best known one dimensional frequency domain design method for polynomial-based filters presented in [2] in each dimension. The frequency domain requirements, such as passband ripple, stopband attenuation, passband and stopband edges are defined independently for each dimension. Furthermore, the filter lengths and polynomial order are also independently determined.

A. Review of the minimax design method

To this end, we assume a lowpass signal in each dimension $x_i(n)\leftrightarrow X_i(e^{j\Omega in})$. A sampling rate in dimension $iF_{in}^{\ i}$ shall be converted by an arbitrary ration according to $F_{out}^{\ i}=R_iF_{in}^{\ i}$ yielding $y_i(l)\leftrightarrow Y_i(e^{j\Omega out})$. In case of R_i>1 (R_i<1) the system realizes interpolation (decimation). The ultimate aim is to determine a continuous-time, finite-length impulse response $h_a^{\ i}(t)$ in dimension i of the sampling rate conversion system such that the following requirements are met:

1. Preserve the usable spectral content of the input signal for

$$\left|Y_i(e^{j\Omega_{out}})\right|\approx\left|X_i(e^{j\Omega_{in}})\right|\forall\left|f_i\right|<F_i/2. \qquad (3)$$

2. Reject in the stopband all spectral components that give rise to aliasing (imaging) as a result of sampling with F_i as much as possible

3. Reject in the stopband all spectral images of $X_i(e^{j\Omega in})$ as much as necessary.

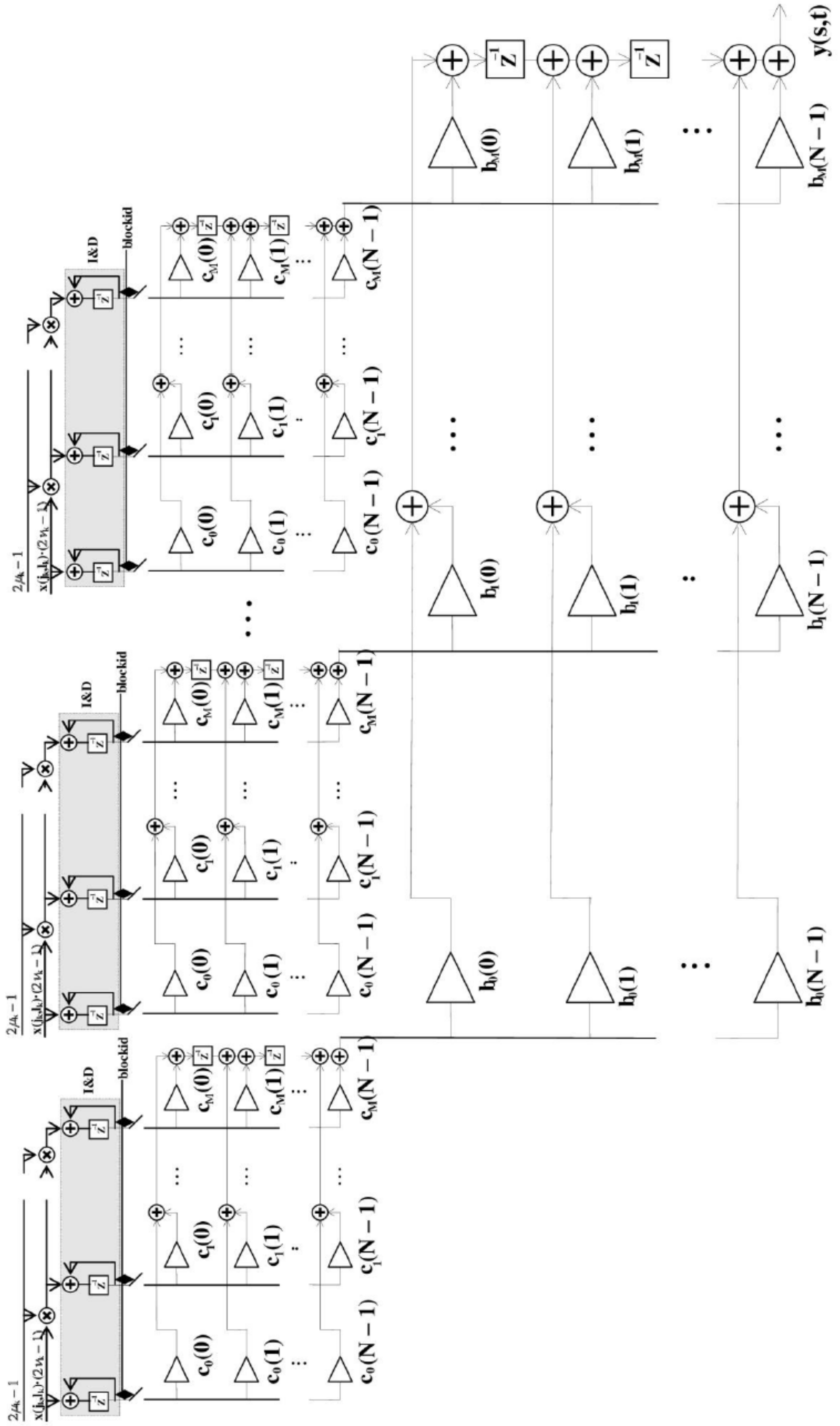

Fig. 4 Two-dimensional Farrow structure using expanden form of structure shown in Fig 3.

It should be pointed out that, in the decimation case, the filter with the impulse response hai(t) acts as an anti-aliasing filter rejecting the frequency components aliasing onto the base-band. On the other hand, in the interpolation case, the role of this filter is to preserve the original base-band region and to eliminate the imaging components. Hence, it acts as an antiimaging filter.

In order to generate a realizable overall system, the criteria for the Fourier transform of $h_a^i(t)$ in each dimension are stated as follows [2]:

$$(1-\delta_p^i) \le H_a^i(f) \le (1+\delta_p) \quad for \quad |f| \le f_p^i$$
$$\left|H_a^i(f)\right| \le \delta_s^i \quad for \quad |f| \in \Phi_s^i, \tag{4}$$

where

$$\Phi_s^i = \begin{cases} [F_i/2, \infty] & for \quad Case \quad A \\ \bigcup_{k=1}^{\infty} [kF_i - f_p^i, kF_i + f_p^i] & for \quad Case \quad B \\ [F_i - f_p^i, \infty] & for \quad Case \quad C \end{cases} \tag{5}$$

In all three cases, the signal is preserved according to the given tolerance in the passband region [0, f_p^i]. Furthermore, the aliasing components are attenuated in the defined manner. In Case A, all components aliasing into the baseband [0, $F_i/2$] are attenuated. In Case B, all components aliasing into the passband [0, f_p^i] are attenuated, but aliasing is allowed in the transition band [f_p^i, $F_i/2$]. In Case C, aliasing into the transition band [f_p^i, $F_i/2$] is allowed only from the band [$F_i/2$, $F_i+f_p^i$]. In the above discussion and in (4) and (5) F_i stands for F_{out}in a decimation case, and F_{in} in an interpolation case in dimension i.

The minimax optimization method introduced in [2] is probably the most convenient and the most flexible solution for designing polynomial-based interpolation filters. This method is applied two times for each dimension independently.

Minimax Optimization Problem: Given N_i, M_i, and a compact subset $\Phi^i \subset [0,\infty)$ as well as a desired function $D_i(f)$ being continuous for $f \in \Phi^i$ and a weight function $W_i(f)$ being positive for $f \in \Phi^i$, find the $(M_i+1)N_i/2$ unknown coefficients $c_m(n)$ ($b_m(n)$ in other dimension) to minimize

$$\delta_\infty^i = \max_{f \in \Phi^i} \left|W_i(f)\left[H_a^i(f) - D_i(f)\right]\right| \tag{6}$$

subject to the given time-domain conditions of $h_a^i(t)$. Here, $H_a^i(f)$ is the real-valued frequency response and $D_i(f)$ is the desired function according to specifications. (For details refer to [2])

The minimax design method has several design parameters. First of all, the design parameters include passband and stopband regions Φ_p^i and Φ_s^i, which are determined with passband f_p^i and stopband f_s^iedges. The desired filter may have several passbands and stopbands as stated in [2]. Next, the minimum stopband attenuation δ_s^i, and maximum allowable passband ripple δ_p^i are also included. Other design parameters are the number of polynomial segments N_i and the order M_i of the polynomial, which determine the number of multipliers in the overall structure, see (2). Finally, some weighting function $W_i(f)$ can be used to give different weights to passband and stopband.

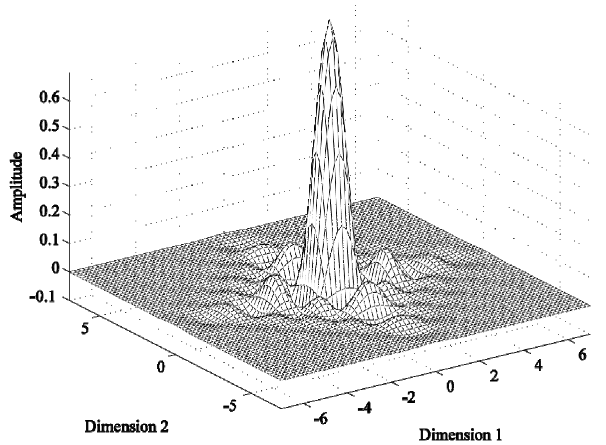

Fig. 5 Impulse response of two-dimensional Farrow structure. The degree of the polynomial in each dimension M_i equals four, and the number of intervals N_i equals 14.

IV. Design Example

In this section we design a separable two-dimensional transposed Farrow structure using frequency domain design method of [2] explained in the previous section. To illustrate this, the following specifications are considered:

Case C *specifications*: The passband and stopband edges are at f_p^i=0.4*Fout*and at f_s^i=0.6*Fout*, in each dimension. Both filters have been designed in minimax sense with the passband weighting equal to unity and stopband weightings of W_i=100, with maximum allowable passband ripple δ_p^i=0.1 and minimum stopband attenuation δ_s^i=0.001 (60dB) in each dimension. The degree of the polynomial in each dimension M_i equals four, and the number of intervals N_i equals 14. Recall that $N_i$is an even integer. Figure 5 illustrates obtained impulse response, and Fig. 6 displays magnitude response of the separable two-dimensional transposed Farrow structure. The complexity of the overall 2-D transposed structure measured by number of multipliers according to (2) is 78.

V. Conclusion

In this paper, we have analyzed implementation structure for the separable two-dimensional transposed Farrow structure. We applied the frequency domain design method of [2] to the two-dimensional case. We have showed that all frequency domain requirements, such as passband ripple, stopband attenuation, passband and stopband edges are defined independently for each dimension. The filter lengths and polynomial order are also independently determined. In this way, it is possible to treat 2-D signal independently in each dimension.

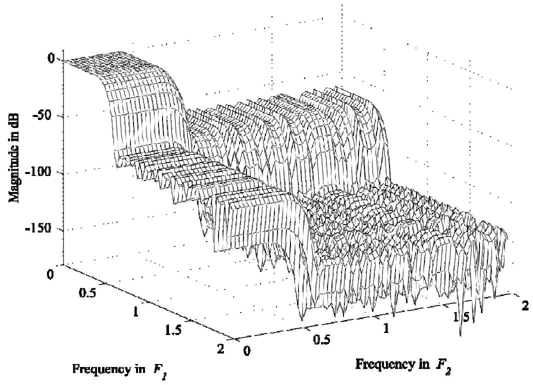

Fig. 6 Frequency response of two-dimensional Farrow structure. The degree of the polynomial in each dimension Mi equals four, and the number of intervals Ni equals 14.

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