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Sliding Mode Fuzzy Control for Rotary Inverted Pendulum

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***Abstract* — A Rotary Inverted Pendulum control is a common problem due to the complexity of the task – inherent nonlinearity and external environmental disturbances. In this paper we design and use the sliding mode controller (SMC) in an attempt to optimally control an Inverted Pendulum during stabilization near the balancing state. Taking into consideration the occurrence of chattering due to implementation imperfections as one of the weaknesses of the SMC, we additionally introduce a fuzzy system to the classical SMC in order to improve control and to avoid drastic changes of the manipulated variable. The numerical simulation and experimental results are analysed in the conclusion.**

***Keywords* — Sliding Mode Control (SMC), Fuzzy Sliding Mode Control (FSMC), Rotary Inverted Pendulum, Optimal Control, Simulation**

I. INTRODUCTION

The Furuta pendulum was invented in 1992 at the Tokyo Institute of Technology. Its inventor, Professor Katsuhisa Furuta was born in Tokyo, Japan, in 1940. The Furuta pendulum [4] is an under-actuated system with 2 degrees of freedom. It’s made of a pendulum attached to the end of a motor-driven link which rotates in the horizontal plane, while the pendulum rotates in the vertical plane perpendicular to the driven link.

We selected this system because it has become a classic for the application of linear and non-linear control theory.

The purpose of this system is to achieve and maintain the pendulum in an upright position by applying torque to the horizontal link. Since the pendulum is a nonlinear system, we use the sliding mode controller to alter its dynamics and stay on the referential value curve. At least, that is the case ideally and only in theory. In practice, sliding mode control approximates this theoretical behavior with a high-frequency switching control signal. This signal causes the system to “chatter” in a tight vicinity of the sliding surface we are trying to follow. In order to optimally control the system – and to try and remedy this - we use the sliding mode controller with the additional introduction of fuzzy control system which deals with the chattering issue that occurs due to the implementation imperfections.

II. INTRODUCTION

QNET rotary Pendulum Trainer is used for experimental procedures and lab exercise. These processes are called Quanser Engineering Trainers for NI ELVIS, or QNET for short.

The simple Modelling VI is shown in Figure 1. It runs the DC motor connected to the pendulum arm in open-loop and plots and corresponding pendulum arm and link angle as well as the applied input motor voltage.

Fig. 1 – QNET Rotary Pendulum Trainer

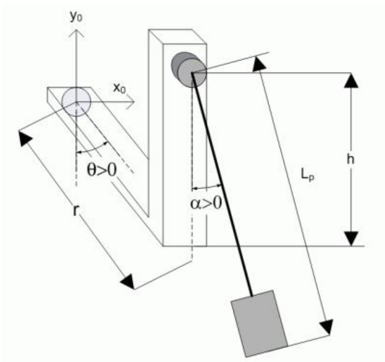

Fig. 2 – Rotary Pendulum System

The QNET-ROTPEN Trainer system consists of a 24-Volt DC motor that is coupled with an encoder and is mounted vertically in the metal chamber. The L-shaped arm, or hub, is connected to the motor shaft and pivots between ±180 degrees. At the end of the arm, there is a suspended pendulum attached. The pendulum angle is measured by an encoder. The source of flow is battery current with change regulator and nonlinear characteristic.

The ROTPEN plant is free to move in two rotary directions. Thus it is a two degree of freedom, or 2 DOF, system. As described in Figure 2, the arm rotates about the Y_0 axis and its angle is denoted by the symbol θ while the pendulum attached to the arm rotates about its pivot and its angle is called α. The shaft of the DC motor is connected to the arm pivot and the input voltage of the motor is the control variable.

In the inverted pendulum experiment, the pendulum angle, α, is defined to be positive when it rotates counter-clockwise. That is, as the arm moves in the positive clockwise direction, the inverted pendulum moves clockwise (i.e. the suspended pendulum moves counter-clockwise) and that is defined as α>0. Recall that in the gantry device, when the arm rotates in the positive clockwise direction the pendulum moves clockwise, which in turn is defined as being positive.

III. Mathematical Modelling of Dynamic Nonlinear Systems

The linear equations of motion [2] of the system are found by linearizing the nonlinear equations of motions, or EOMs, presented above about the operation point α = π and solving for the acceleration of the terms θ and α. For the state

$$x = [x_1, x_2, x_3, x_4]^T$$

$$x_1 = \theta$$

$$x_2 = \alpha \qquad (1)$$

$$x_3 = \frac{\partial}{\partial t}\theta$$

$$x_4 = \frac{\partial}{\partial t}\alpha$$

the linear state-space representation of the ROTPEN Inverted Pendulum is

$$\frac{d}{dt}x(t) = Ax(t) + Bu(x)$$
$$y(t) = Cx(t) + Du(x) \qquad (2)$$

where u(x) = V_m and the A, B, C, and D matrices are

$$A = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{rM_p^2 l_p^2 g}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & -\frac{K_t K_m (J_p + M_p l_p^2}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \\ 0 & -\frac{M_p l_p g (J_{eq} + M_p r^2)}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & \frac{M_p l_p K_t r K_m}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \end{vmatrix}$$

$$B = \begin{vmatrix} 0 \\ 0 \\ \frac{K_t (J_p + M_p l_p^2)}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \\ -\frac{M_p l_p K_t r}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \end{vmatrix}$$

$$C = \begin{vmatrix} \theta & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & \alpha \end{vmatrix}$$

$$D = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

Finally,

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{rM_p^2 l_p^2 g}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & -\frac{K_t K_m (J_p + M_p l_p^2}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \\ 0 & -\frac{M_p l_p g (J_{eq} + M_p r^2)}{J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2} & \frac{M_p l_p K_t r K_m}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} & 0 \end{vmatrix} x + \begin{vmatrix} 0 \\ 0 \\ \frac{K_t (J_p + M_p l_p^2)}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \\ -\frac{M_p l_p K_t r}{(J_p J_{eq} + M_p l_p^2 J_{eq} + J_p M_p r^2) R_m} \end{vmatrix} u \qquad (3)$$

IV. Sliding Mode Fuzzy Control

In sliding mode fuzzy control [1], [3] the nonlinear autonomous open loop model is given in the standard form

$$\dot{x} = f(x, u) \qquad (4)$$

The SMFLC is given as a set of fuzzy rules, derived directly from the above nonlinear model, each fuzzy rule being of the form

R_c^i: if s=LS^i then u=LU^i,

or the form

R_c^i: if s= LS^i and d=LD^i then u=LU^i,

where s is the distance between the state vector and the sliding surface, and d is the distance between the state vector and the normal vector to the sliding surface where the normal vector goes through the origin of the state space. Furthermore, $LS^i \in$ TS and $LD^i \in$ TD are the fuzzy values of the fuzzy state variables s and d in the i-th fuzzy region of the fuzzy state space; LU^i is the fuzzy input vector corresponding to the i-th fuzzy region of the fuzzy state space; TS, TD and TU are the term-sets of s, d, and u containing the range of fuzzy values of s, d, and u.

This type of FLC is an extension of crisp sliding mode control, and crisp sliding mode control with boundary layer. The control law of the SMFLC has a static transfer characteristic reflecting the relationship between the variables s and d and the input u.

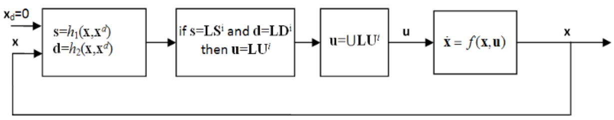

Fig. 3 – Sliding Mode FLC

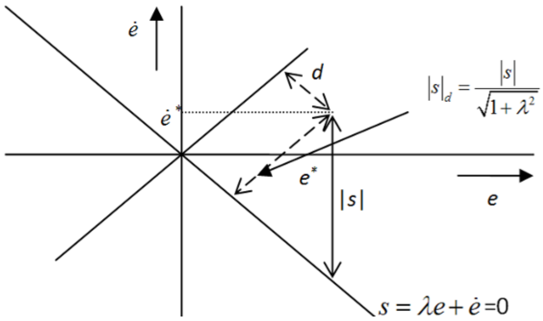

Fig. 4 – Graphical Interpretation of s and d

The magnitude of a specific positive/negative fuzzy values of u is determined on the basis of the distance |s| between its corresponding state vector e and the sliding line s = λe + ė = 0. This is normally done in such a way that the absolute value of the required input u increases/decreases with the increasing/decreasing distance between the state vector e and the sliding line s = 0.

SMC with BL provides a linear transfer characteristic with lower and upper bounds while the transfer characteristic of an SMFLC is not necessarily a straight line between these bounds, but a curve that can be adjusted to reflect given performance requirements. For example, normally a fast rise time and as little overshoots as possible are the required performance characteristics for the closed loop system. These can be achieved by making the controller gains much larger for state space regions far from the sliding line than its gains in state space regions close to the sliding line.

In this connection it has to be emphasized that an SMFLC is a state dependent filter. The slope of its transfer characteristic decides the convergence rate to the sliding line and, at the same time, the bandwidth of the unmodeled disturbances that can be coped with. This means that far from the sliding line higher frequencies are allowed to pass through than in the neighborhood of it. The other function of this state dependent filter is given by the sliding line itself. That is, the velocity with which the origin is approached is determined by the slope λ of the sliding line s = 0.

Because of the special form of the rule base of a diagonal form FLC each fuzzy rule can be redefined in terms of the fuzzy value of the distance |s| between the state vector e and the sliding line, and the fuzzy value of the input u corresponding to this distance. This helps to reduce the number of fuzzy rules especially in the case of higher order systems. Despite the advantages of an SMFLC it poses a number of problems the solutions of which can improve its performance and robustness.

V. Sliding Mode Fuzzy Control of Rotary Inverted Pendulum

For the sliding line s = λe + ė, where $e = \theta_1 - \theta_1^{ž}$, we can introduce a total law of control

$$u = -\lambda \dot{e} - K_{fuzz}(s_p, d)\,\mathrm{sgn}(s) \qquad (5)$$

where

$$s_p = \frac{s}{\sqrt{1+\lambda^2}}. \qquad (6)$$

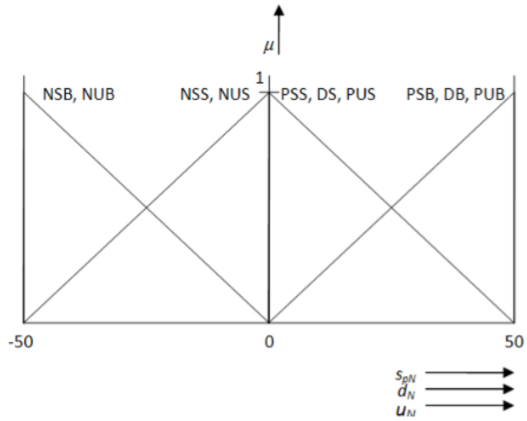

Fig. 5 – Fuzzy Strings

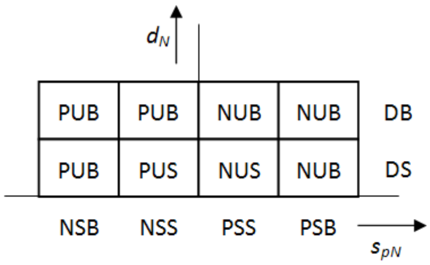

Fig. 6 – Dependency Diagram for u_N of s_{pN} and d_N

In a normalized phase plane, "distances" S_{pN} and d_N are

$$S_{pN} = \frac{e_N + \dot{e}_N}{\sqrt{2}} \qquad (7)$$

and

$$d_N = \left| \frac{-e_N + \dot{e}_N}{\sqrt{2}} \right|. \qquad (8)$$

For normalized values S_{pN} and d_N and normalized control variable u_N, the corresponding fuzzy strings are defined (Fig. 5). Dependency diagram for u_N of S_{pN} and d_N is shown on Fig. 6, from which the fuzzy rules for calculation of normalized output of the controller u_N can clearly be seen.

The rule for choosing λ is that $\lambda \ll V_{su}$ where V_{su} represents non-modelled frequencies. If we assume that non-modelled frequencies are much larger than the natural frequency of the system, we only need to choose $\lambda \leq \omega$. Based on correlation tests, we can determine the normalization factors $N_{\dot{e}}$ and N_e, and based on the upper limit $K_{fuzz}|max$ and the corresponding normalization factor N_u.

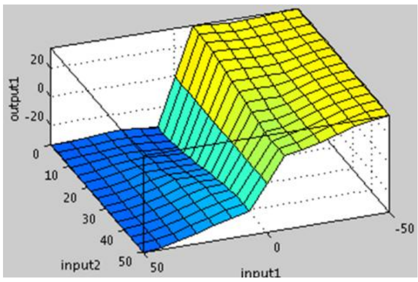

Fig. 7 – Fuzzy Rules Control Surface

On Figure 7, a fuzzy rules control surface is shown. Values s_{pN} and d_N represent inputs into the fuzzy controller, while the normalized amplifying factor K_{fuzz} represents its output.

The response of the closed system before and after adjustment of the normalization and denormalization factors, as well as amplification, are shown on Figures 8 and 9 which evidently show that system performances can be tweaked through scaling in order for Fuzzy Logical Controller to complete the requested tasks.

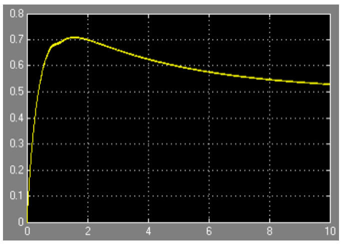

Fig. 8

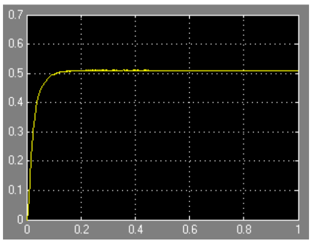

Fig. 9

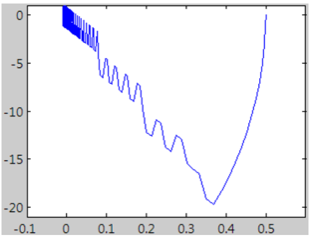

Fig. 10 – Dependency Diagram of Error e and Error Correction de/dt

Figure 10 shows a dependency diagram of error e and error correction de/dt. Obviously, when the dependency curve once enters the border area around the sliding line

$$S = \lambda e + \dot{e} \tag{9}$$

it stays in that area, meaning that the pendulum is balanced.

CONCLUSIONS

Computer intelligence is set on finding the approximate solutions for practical problems that have been defined in advance, where the techniques of training, theories of control and artificial intelligence become significantly mutually compatible. The application of models based on elements of computer intelligence has shown significant advantages with complex systems.

In this paper we have designed and used the Sliding Mode Controller to optimally control the Inverted Pendulum during stabilization near its balancing state. In order to further improve control and at the same time avoid any drastic changes of the manipulated variable, we have additionally introduced the Fuzzy System into the classical SMC.

This approach, based on Sliding Mode Fuzzy Control, makes it possible to ensure quality control which as a consequence has significant loss of chattering and significant overall performance improvement of the system which, by adjusting of the normalization and denormalization factors, as well as amplification can be additionally improved.

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